Solution

Partial solution (27 points)

Let the binary representations of x and y be different at the i-th digit (from the right). Transmit $h = 2i$ if the i-th digit of x is 0 and $h = 2i - 1$ if the i-th digit of x is 1.

B should answer "yes" iff the $\left\lfloor \frac{h+1}{2} \right\rfloor$-th digit of f is $h \mod 2$.

Partial solution (60 points)

Let h be the smallest i such that the i-th digit of x is 0 and the i-th digit of y is 1 if such a number i exists. Otherwise, the digit sum d_x of x is obviously larger than the digit sum d_y of y. Then, you can transmit $\lceil \log_2 N \rceil + i$ where i is any number such that the i-th digit of d_x is 1 and the i-th digit of d_y is 0.

If $h \leq \lceil \log_2 N \rceil$, B has to check whether the h-th digit of q is 0. Otherwise, he must check whether the $(h - \lceil \log_2 N \rceil)$-th digit of the digit sum of q is 1.

Full solution

Let K be the maximum number h that A is allowed to shout over to B.

Assume A and B aggreed on some sets $M_i \subseteq \{1, \ldots, K\}$ for $1 \leq i \leq N$.

Let B answer "yes" if and only if $h \in M_q$. Then, A can simply shout any number $h \in M_x \setminus M_y$ over to B, provided that M_x is not a subset of M_y. Hence, this strategy works out if no set M_i is a subset of any other set M_j.

We can simply choose M_i to be pairwise distinct $\lfloor K/2 \rfloor$-element subsets of $\{1, \ldots, K\}$.

Remark. This choice of the M_i is optimal, i.e., the problem is solvable if and only if $\binom{K}{\lfloor K/2 \rfloor} \geq N$ (cf. Sperner's theorem).